JOURNAL OF MODERN OPTICS, 15 APRIL–10 MAY 2004
VOL. 51, NO. 6–7, 1105–1106

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Short communication
Localized states (qubits), entanglement and decoherence from Wigner zoo

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(*Received 2 November 2003*)

We present the application of the variational-wavelet analysis to the calculations and analysis of the solutions of Wigner/von Neumann/Moyal and related equations corresponding to the nonlinear (polynomial) dynamical problems [1, 2]. (Naive) deformation quantization, the multiresolution representations and the variational approach are the key points. We construct the solutions via the multiscale expansions in the high-localized nonlinear eigenmodes in the base of the compactly supported wavelets and the wavelet packets. We demonstrate the appearance of (stable) localized patterns (waveletons) and consider entanglement and decoherence as possible applications. Our goals are some attempt of classification and the explicit numerical-analytical constructions of the existing zoo of possible/realizable quantum states. There is a hope on the understanding of relation between the structure of initial Hamiltonians and the possible types of quantum states and the qualitative type of their behaviour. Inside the full spectrum there are at least four possibilities which are the most important from our point of view for possible realization of quantum-like computations: localized states, chaotic-like or/and entangled patterns, localized (stable) patterns (waveletons). We consider the calculations of the Wigner functions $W(p, q, t)$ (WF) corresponding to the classical polynomial Hamiltonians $H(p, q, t)$ as the solution of the Wigner–von Neumann equation:

$$i\hbar \frac{\partial}{\partial t} W = H * W - W * H \tag{1}$$

and related Wigner-like equations, e.g. the Wigner transform of the master equation (Lindblad-like) describing the decoherence

$$\dot{W} = \{H, W\} + \sum_{n \geq 1} \frac{\hbar^{2n}(-1)^n}{2^{2n}(2n+1)!} \partial_x^{2n+1} U(x) \partial_p^{2n+1} W + 2\gamma \partial_p p W + D \partial_p^2 W \tag{2}$$

We have constructed the following quantum states: localized (as a model for qubit), entangled patterns, localized (stable) patterns (e.g. modelling the decoherence as a result of interaction with the environment). The obtained solutions of these equations have the following multiscale or multiresolution decomposition

Journal of Modern Optics ISSN 0950–0340 print/ISSN 1362–3044 online
http://www.tandf.co.uk/journals
DOI: 10.1080/09500340410001664098

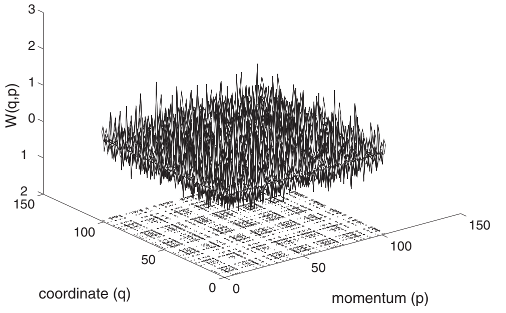

Figure 1. Entangled-like Wigner function.

via nonlinear high-localized eigenmodes

$$W(t, x_1, x_2, \ldots) = \sum_{(i,j)\in Z^2} a_{ij} U^i \otimes V^j(t, x_1, x_2, \ldots),$$
$$V^j(t) = V_N^{j,\,\mathrm{slow}}(t) + \sum_{l\geq N} V_l^j(\omega_l t), \quad \omega_l \sim 2^l\ ,$$
$$U^i(x_s) = U_M^{i,\,\mathrm{slow}}(x_s) + \sum_{m\geq M} U_m^i(k_m^s x_s), \quad k_m^s \sim 2^m\ , \tag{3}$$

which correspond to the full multiresolution expansion in all underlying phase space or space–time scales. Formulae (3) give the expansion into a slow part and fast oscillating parts for arbitrary N, M. So, we may move from the coarse scales of resolution to the finest ones for obtaining more detailed information about the dynamical process. In this way one obtains contributions to the full solution from each scale of resolution or each time/space scale or from each nonlinear eigenmode. It should be noted that such representations give the best possible localization properties in the corresponding (phase)space/time coordinates. Formulae (3) do not use perturbation techniques or linearization procedures. The modelling demonstrates the appearance of different (stable) pattern formation from high-localized coherent structures or chaotic/entangled behaviour. Our (nonlinear) eigenmodes are more realistic for the modelling of nonlinear classical/quantum dynamical process than the corresponding linear Gaussian-like coherent states. Here we mention only the best convergence properties of the expansions based on wavelet packets, which realize the minimal Shannon entropy property and the exponential control of convergence of expansions like (3). As an example, waveletons correspond to the result of superselection (einselection) after decoherence process started from entanglement state, demonstrated by figure 1 and constructed from localized qubit-like states [2]. It should be noted that we can control the type of behaviour on the level of the reduced algebraic system [2].

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